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Anomaly mediation in supergravity theories

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Abstract: We consider the effects of anomalies on the supersymmetry-breaking parameters in supergravity theories. We construct a supersymmetric expression for the anomaly-induced terms in the 1PI effective action; we use this result to compute the complete one-loop formula for the anomaly-induced gaugino mass. The mass receives contributions from the super-Weyl, Kähler, and sigma-model anomalies of the supergravity theory. We point out that the anomaly-mediated gaugino mass can be affected by local counterterms that cancel the super-Weyl-Kähler anomaly. This implies that the gaugino mass cannot be predicted unless the full high-energy theory is known.

Keywords: Anomalies in Field and String Theories, Supersymmetry Breaking, Supergravity Models, Supersymmetric Effective Theories.

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1. Introduction and summary

Supersymmetry at the electroweak scale is an appealing possibility for physics beyond the standard model. Not only does it render the ratio of the electroweak scale to the Planck scale, $M_W/M_{\mathrm{P}} \sim 10^{-17}$, stable against quadratically divergent radiative corrections, but it can also explain the size of the hierarchy if supersymmetry is broken by some nonperturbative mechanism.

It is, therefore, an urgent and open problem to find a viable and testable method for dynamically breaking supersymmetry and generating weak-scale masses for the superpartners of the known elementary particles. The problem has two aspects, which are not necessarily distinct. First, one has to dynamically break supersymmetry, and second, one has to communicate the supersymmetry breaking to the visible-sector particles. The main ideas for this are supergravity and gauge mediation.

In this paper we will consider supergravity-mediated supersymmetry breaking. In such theories, the communication of supersymmetry breaking to the scalar fields is almost automatic. For a generic Kähler potential, once supersymmetry is broken in a hidden sector, visible-sector scalars gain mass as a consequence of the tree-level supergravity equations of motion. The scale of the mass, M_0, is set by dimensional analysis: $M_0 \sim M_{\mathrm{SUSY}}^2/M_{\mathrm{P}} \sim m_{3/2}$, where M_{SUSY} is the scale of supersymmetry breaking and $m_{3/2}$ is the gravitino mass. If M_{SUSY} is of order an intermediate scale, $M_{\mathrm{SUSY}} \sim 10^{10}\,\mathrm{GeV}$, the soft scalar masses are of order M_W.

Gaugino masses, as well as A-terms, are more difficult to obtain because they break a $\mathrm{U}(1)_R$ symmetry. The simplest way to generate these terms is to include an effective gauge-singlet chiral superfield in the low-energy effective lagrangian. The problem is that such singlets are not necessarily present in generic models of supersymmetry breaking. Moreover, fundamental singlets can cause cosmological difficulties, or destabilize the gauge hierarchy through quadratically divergent radiative corrections. It is important, therefore, to fully investigate alternative mechanisms for generating gaugino masses.

It was recently pointed out by Randall and Sundrum [1], and Giudice, Luty, Murayama, and Rattazzi [2], that once supersymmetry is broken in the hidden sector of a theory, gaugino masses and A-terms are generated at one loop from anomaly-related graphs. This is an important result because it shows that these terms are automatically present, even in theories *without* gauge singlets. The simplest way to understand their result is to consider the one-loop renormalization of the visible-sector gauge couplings,[1,2]

$$\frac{1}{4}\int d^2\theta\left(1-\frac{g^2 b_0}{16\pi^2}\log\frac{\Lambda^2}{\Box}\right)W^\alpha W_\alpha+\text{h.c.}\,, \tag{1.1}$$

where b_0 is the first coefficient of the beta function and Λ is the ultraviolet cutoff. As pointed out in refs. [1, 2], the leading supersymmetry-breaking effect can be found, in the absence of Planck-scale hidden-sector expectation values, by replacing the ultraviolet cutoff in (1.1) with a supersymmetry-breaking spurion superfield,

$$\Lambda\to\Lambda\,\exp(m_{3/2}\,\theta^2)\,. \tag{1.2}$$

This leads, after substitution in (1.1), to the following "anomaly-induced" gaugino mass,

$$m_{1/2}=-\frac{g^2 b_0}{16\pi^2}\,m_{3/2}\,. \tag{1.3}$$

One-loop A-terms are induced in a similar way. This mechanism for generating gaugino masses gives rise to an exciting new phenomenology [4].

In what follows we will take a closer look at the physics that underlies anomaly mediation, in the full supergravity context. Our starting point will be the general supergravity lagrangian. The classical symmetries of this lagrangian are super-Weyl-Kähler invariance (or, in components, simply Kähler invariance), as well as various global symmetries that act as sigma-model isometries. These symmetries induce anomalous chiral rotations on the fermions. We shall see that anomaly mediation results from quantum anomalies in these classical symmetries.

In section 2 we will elaborate on these points, and present the terms in the 1PI effective action that are induced by the chiral anomalies associated with the

[1] In this paper, we omit the index for the adjoint representation for the gauge multiplets. For non-abelian gauge groups, $W^\alpha W_\alpha$ and $F_{mn}F^{mn}$ should be understood as $W^{(a)\alpha}W^{(a)}_\alpha$ and $F^{(a)}_{mn}F^{(a)mn}$, where (a) is the index of the adjoint representation. We use the conventions of ref. [3].

[2] For simplicity, we do not include gauge kinetic functions. In general, gauge kinetic functions can occur at the tree or loop level. If present, they can give non-anomalous contributions to gaugino masses. We are interested in anomaly-induced gaugino masses, so we do not consider them here.

super-Weyl-Kähler and sigma-model symmetries. These nonlocal terms are entirely determined by Bose symmetry, unitarity, analyticity, and gauge invariance. They are, therefore, independent of the choice of regulator[3] [5]. By demanding local supersymmetry, we will find the unique superspace extension of these nonlocal anomaly-induced terms [6].

In section 3 we will discuss the conditions under which these nonlocal terms are present in the effective action. We will argue that they are indeed present in the case of interest. We will then derive the general formula for the one-loop anomaly-induced contribution to the gaugino mass, valid for hidden-sector models with arbitrary expectation values,

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[(3T_G - T_R)\, m_{3/2} + (T_G - T_R)\, K_i F^i + \frac{2\, T_R}{d_R}(\log\det K|_R{}'')_{,i}\, F^i\right]. \tag{1.4}$$

In this expression, $3T_G - T_R$ is the beta function, b_0, where T_G is the Dynkin index of the adjoint representation, normalized to N for $\mathrm{SU}(N)$, and T_R is the Dynkin index associated with the representation R of dimension d_R, normalized to $1/2$ for the $\mathrm{SU}(N)$ fundamental. A sum over all matter representations is understood. The second and third terms in this expression involve the Kähler potential, K, and the expectation values of the auxiliary fields, F^i, evaluated in the Einstein frame. This result for $m_{1/2}$ generalizes the one given in refs. [1, 2], and reduces to it when all expectation values are much less than M_{P}.

In section 4 we will see that anomaly-induced gaugino masses can be modified by contributions from Planck-scale physics. In general, we expect such contributions because supergravity is a nonrenormalizable, effective field theory, valid for energies much less than M_{P}. In string theory, for example, the heavy string modes can induce anomalous Green-Schwarz-like terms in the effective action. We shall see that these terms can contribute to the gaugino masses. This illustrates that the gaugino masses can be significantly modified — or canceled altogether — by high energy contributions that are outside the realm of the effective supergravity theory.

2. Anomalies in matter-coupled supergravity

In this section, we will define the classical symmetries of matter-coupled supergravity: super-Weyl-Kähler invariance and sigma-model isometries. We will compute their quantum anomalies and write down, following Cardoso and Ovrut [6], the terms they induce in the 1PI effective action. We will restrict our attention to the mixed super-Weyl-Kähler gauge and sigma-model gauge anomalies because they are the anomalies that are relevant for gaugino masses.

[3] A 1PI argument for the regulator independence of anomaly mediation, based on local supersymmetry, was also given in ref. [2].

The super-Weyl-Kähler and sigma-model symmetries of classical supergravity lagrangian can be given a full superspace description (see [3]). However, for the case at hand, it is perhaps simpler to study them through their action on the fermions in the supergravity lagrangian. To this end, let us consider the kinetic terms for the fermions:

$$\mathcal{L}_{\rm kin} = -iK_{ij^*}\,\bar\chi^j\bar\sigma^a\left[\left\{\partial_a + \frac{i}{6}\,b_a - \frac{1}{6}\left(K_j\partial_a A^j - K_{j^*}\partial_a A^{*j}\right)\right\}\chi^i + \Gamma^i_{jk}\partial_a A^j\chi^k\right] - \\ - i\bar\lambda\bar\sigma^a\left(\partial_a - \frac{i}{2}b_a\right)\lambda\,, \tag{2.1}$$

where A^i and χ^i are the scalar and fermionic components of i-th chiral superfield, the λ are the gauginos, and b_a is the auxiliary vector field in the supergravity multiplet. Furthermore, let us define $K_i \equiv \partial K/\partial A^i$, and K_{ij^*} and Γ^i_{jk} to be the Kähler metric and Kähler connection, respectively,

$$K_{ij^*} = \frac{\partial^2 K}{\partial A^i\partial A^{*j}}\,,\qquad K_{\ell j^*}\Gamma^\ell_{ik} = \frac{\partial K_{ij^*}}{\partial A^k}\,. \tag{2.2}$$

We choose to write eq. (2.1) without eliminating the auxiliary field b_a because we wish to preserve off-shell supersymmetry. We assume that the gauge fields couple in the usual manner.

The fermion kinetic terms (2.1) contain connections for three local symmetries. The first is a $\mathrm{U}(1)_R$ symmetry that is part of the superconformal group: it acts on the gauginos, the matter fermions, and the supergravity auxiliary field as follows,

$$\begin{aligned} \lambda &\to e^{i\alpha/2}\,\lambda \\ \chi &\to e^{-i\alpha/6}\,\chi \\ b_a &\to b_a\,+\,\partial_a\alpha\,. \end{aligned} \tag{2.3}$$

The connection b_a shifts under the $\mathrm{U}(1)_R$ symmetry because it is the gauge field of superconformal supergravity. (Recall that the action and auxiliary field structure of $N=1$ Einstein supergravity can be obtained by gauge fixing the superconformal supergravity action; see e.g. [7].) From these transformations we see that the chiral anomaly associated with the superconformal symmetry is proportional to $b_0 = 3T_G - T_R$.

The second symmetry is a local $\mathrm{U}(1)_K$ symmetry that acts on the matter fermions but not the gauginos. It is compensated by a shift of the Kähler connection:

$$\begin{aligned} \chi &\to e^{-i\beta/6}\,\chi \\ i(K_m\partial_a A^m - K_{m^*}\partial_a A^{*m}) &\to i(K_m\partial_a A^m - K_{m^*}\partial_a A^{*m})\,+\,\partial_a\beta\,. \end{aligned} \tag{2.4}$$

The chiral anomaly of this symmetry is proportional to T_R. Note that the $\mathrm{U}(1)_R\times\mathrm{U}(1)_K$ symmetry of the fermion kinetic terms is explicitly broken by other terms in the supergravity action. The unbroken symmetry is super-Weyl-Kähler symmetry, which in component form is usually called Kähler invariance [3]. This, however, does not affect the nonlocal anomaly terms that are induced in the 1PI effective action.

The third connection entering the fermion kinetic terms is the sigma-model connection, $\Gamma^i_{jk} \equiv K^{i\ell^*}\partial_j K_{k\ell^*}$, where $K^{i\ell^*}$ is the inverse Kähler metric. It acts as a $\mathrm{U}(N)$ connection on the matter fermions, where N is the number of chiral multiplets in the theory. However, only the U(1) subgroup contributes to the gaugino masses. Therefore, in what follows, we restrict our attention to this U(1), and write

$$\Gamma^i_{jk}\partial_a A^j = d_R^{-1}\,\delta^i_k \Gamma^\ell_{j\ell}\partial_a A^j + \cdots = d_R^{-1}\,\delta^i_k\,(\log\det K|_R{}'')_{,j}\partial_a A^j + \cdots\,, \tag{2.5}$$

where the dots denote the non-singlet part of the sigma-model connection, and $K|_R{}''$ is the Kähler metric restricted to the representation R. In eq. (2.5), a sum over all matter representations R is understood, and the sum over ℓ is restricted to the appropriate representations.

The super-Weyl, Kähler and sigma model symmetries are all anomalous. The connections b_a, $K_m\partial_a A^m - K_{m^*}\partial_a A^{*m}$, and $\Gamma^i_{jk}\partial_a A^j$ can be viewed as background gauge fields coupled to anomalous currents. Consider, for example, the connection b_a. The gauge triangle diagram gives rise to a nonlocal term in the 1PI effective action, whose form is determined by Bose symmetry, unitarity, analyticity, and gauge invariance [5]:

$$\Delta\mathcal{L} \;=\; \frac{g^2}{96\pi^2}\,(3T_G - T_R)\,\frac{\partial_a b^a}{\Box}\,F_{mn}\tilde{F}^{mn}\,. \tag{2.6}$$

In this expression, F_{mn} and $\tilde{F}^{mn}$ are the field strength and the dual field strength associated with a set of dynamical gauge fields, and b_a is the background gauge field that couples to the anomalous $\mathrm{U}(1)_R$ current. Under a background gauge transformation, $b_a \to b_a + \partial_a\alpha$, $\Delta\mathcal{L}$ has a local variation that expresses the $\mathrm{U}(1)_R$ anomaly. Similar terms appear for the other two currents.

Note that the anomaly (2.6) depends on the quadratic Casimirs T_G and T_R. These Casimirs include only those fermions that are effectively massless at the momenta of interest. If a fermion propagating in the loop has a Dirac mass m, it decouples from the anomaly for external momenta $q^2 \ll m^2$. (The situation for masses generated by spontaneous symmetry breaking will be discussed in section 4.)

In a supersymmetric theory, the above anomalies can be lifted to superspace [3]. Following Cardoso and Ovrut [6], the result is as follows

$$\begin{aligned}\Delta\mathcal{L} = -\frac{g^2}{256\pi^2}\int d^2\Theta\; 2\mathcal{E}\, W^\alpha W_\alpha\,\frac{1}{\Box}\,(\bar{\mathcal{D}}^2 - 8R)\,\times \\ \times\left[4(T_R - 3T_G)\,R^+ - \frac{1}{3}T_R\,\mathcal{D}^2 K + \frac{T_R}{d_R}\mathcal{D}^2\log\det K|_R{}''\right] + \text{h.c.}\end{aligned} \tag{2.7}$$

The first term, which contains the R^+ superfield, arises from the $\mathrm{U}(1)_R$ anomaly. It is proportional to the beta function, $b_0 = 3T_G - T_R$. The second and third terms express the $\mathrm{U}(1)_K$ anomaly and the U(1) piece of the sigma-model anomaly. Together they have the appropriate local variations to produce the quantum anomalies associated with the super-Weyl-Kähler and U(1) sigma-model symmetries [6].

The anomaly for each connection can be obtained by a background superfield calculation of the two-point gauge superfield Green's function, with a single insertion of the appropriate background field (the superspace curvature R for the first term, and K or $\log\det K''$ for the last two terms). Therefore this result should be interpreted as the leading contribution in a background-field expansion. In particular, the inverse laplacian should be evaluated in flat spacetime.

To see that the superspace expression (2.7) does indeed reproduce the correct component anomaly terms, one must expand in components. Consider, for example, the $\mathrm{U}(1)_R$ term. The component expansion of the superfield R^+ contains the following terms [3],

$$R^+ = -\frac{1}{6}\left[M^* + \cdots + \bar{\Theta}^2\left(-\frac{1}{2}\mathcal{R} + ie_a^m\mathcal{D}_m b^a + \cdots\right)\right], \tag{2.8}$$

where $\mathcal{R}$ is the Einstein curvature scalar. If we substitute (2.8) into the first term in (2.7), and take the Θ^2 component of $W^\alpha W_\alpha$ and the lowest component of $(\bar{\mathcal{D}}^2 - 8R)R^+$, we recover the same nonlocal $\mathrm{U}(1)_R$ anomaly as in (2.6).

The superspace expression (2.7) also contains the supersymmetric partners of the $\mathrm{U}(1)_R$ anomaly. In particular, it contains the nonlocal term

$$\Delta\mathcal{L} = \frac{g^2}{192\pi^2}\,(3T_G - T_R)\,\frac{\mathcal{R}}{\Box}\,F_{mn}F^{mn}\,. \tag{2.9}$$

This term arises from a one loop graph with one graviton and two gauge boson vertices. Under a conformal rescaling of the metric, $g_{mn} \to \exp(2\lambda) g_{mn}$, the curvature scalar shifts as $\mathcal{R} \to \mathcal{R} + 6\,\Box\lambda + \mathcal{O}(\lambda^2)$. We see that the nonlocal term (2.9) also expresses the conformal anomaly of the theory.

In the following section, we will compute the full anomaly-mediated gaugino mass term. Before we do that, however, we first discuss the conditions under which eq. (2.7) is valid. We start by noting that our derivation required two essential ingredients:

(i) A nonlocal anomaly term of the type given in (2.6).

(ii) Local supersymmetry of the 1PI action.

The first item is obvious. If there are no anomalies, there are no anomaly-mediated contributions to gaugino masses. For visible-sector gaugino masses, the associated anomaly diagrams contain visible-sector fermions in the loop. The anomaly receives contributions from all fermions whose mass is less than the weak scale, M_W.

The second item says that the superspace expression only holds when the superpartners of the loop fermions are active at the scale of interest. For gaugino masses, the loop contains visible-sector particles. Since all visible-sector superparticles have weak-scale masses, the effective theory is essentially supersymmetric. This implies that the expression (2.7) can indeed be used to extract gaugino masses.

A more refined analysis confirms this intuition. Consider the case of a visible sector in which there are scalar mass terms present at the tree level, but no tree-level fermion masses. Since the fermions are massless, triangle diagrams induce anomalies in the anomalous currents.

Is the Cardoso-Ovrut result valid for this case? Eq. (2.7) depends on graphs that contain just one background-field insertion in the gauge two-point function. Since the scalar mass terms arise from such insertions, one should, in fact, sum over all hidden-sector insertions. These insertions have important effects. For example, they can change the conformal anomaly, eq. (2.9). If the scalars have mass, their contribution must decouple at momenta below their mass. The decoupling of the scalars is not described by the Cardoso-Ovrut term because the extra insertions are not included.

Fortunately, the scalar mass insertions have no effect on the diagrams that give rise to gaugino masses. One way to see this is to note that gaugino masses arise as a result of loops with Pauli-Villars regulator fields. Therefore the anomaly-induced gaugino masses are present whether or not the visible-sector scalar fields have tree-level masses.

By way of contrast, consider now the case of the hidden-sector loops that contribute to the hidden-sector gauge two-point function. Naively, one might think that these loops would generate visible-sector scalar masses through the Cardoso-Ovrut term. However, they do not because conditions (i) and (ii) both fail in the hidden sector. In particular, hidden-sector fermions are generally not massless, so the terms of the form (2.6) are not present at scales below $M_{\rm SUSY}$. Second, even if there are hidden-sector massless fermions, their superpartners are heavy, typically of order $M_{\rm SUSY}$. Therefore the Cardoso-Ovrut term cannot be used to infer one-loop anomaly-induced masses for the visible-sector supersymmetric scalars.

3. Anomaly-induced gaugino masses

In the previous section, we found that the super-Weyl-Kähler and sigma-model anomalies generate nonlocal terms in the 1PI effective action. The terms are induced by vector-field two-point diagrams that contain one additional insertion of a supergravity or hidden-sector background field. In this section we will see that they give rise to gaugino masses when the background fields obtain supersymmetry-breaking expectation values.

The anomaly-induced contribution to the gaugino mass can be easily computed from (2.7). One first extracts the Θ^2 components of the terms multiplying $W^\alpha W_\alpha$.

This gives

$$\frac{g^2}{16\pi^2}\left[\frac{M^*}{3}\left(3T_G-T_R\right)+\frac{2}{3}T_R\,K_iF^i-\frac{2T_R}{d_R}\left(\log\det K|_R{}''\right)_{,i}F^i\right]. \tag{3.1}$$

In this expression, the expectation values of the auxiliary fields M and F^i are evaluated in the general "supergravity frame," in which the Einstein term is of the form $\exp(-K/3)\mathcal{R}$.

To make contact with the usual supergravity lagrangian (see e.g. [3]), one must transform to the "Einstein frame," in which the Einstein term takes its canonical form. The frame transformation is accomplished by a Weyl rescaling of the metric and a redefinition of the fermionic and auxiliary fields. The relevant transformations are as follows [3],

$$\begin{aligned} e_c{}^m &= e^{-2\sigma}\,e'_c{}^m \\ \lambda &= e^{-3\sigma}\,\lambda' \\ M^* &= e^{-2\sigma}\left(M'^* - F'^iK_i\right) \\ F^i &= e^{-2\sigma}\,F'^i\,, \end{aligned} \tag{3.2}$$

where the primed quantities are in the Einstein frame and $\sigma = K/12$. Note that the field M transforms inhomogeneously, while the F^i transforms homogeneously.

Using these results, it is not hard to write the anomaly-induced gaugino mass in the usual Einstein frame. One first redefines the auxiliary fields as in (3.2) and then one drops the primes. This gives the complete one-loop anomaly-induced contribution to the gaugino mass,

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[\left(3T_G-T_R\right)m_{3/2}+\left(T_G-T_R\right)K_iF^i+\frac{2T_R}{d_R}\left(\log\det K|_R{}''\right)_{i}F^i\right]. \tag{3.3}$$

In this expression, we have substituted the Einstein-frame expectation value of M,

$$M^* = -3e^{K/2}\,P^* \equiv -3m_{3/2}\,, \tag{3.4}$$

where P is the superpotential. We also require the field F^i to be evaluated in the Einstein frame, where

$$F^i = -e^{K/2}\,K^{ij^*}\left(P^*_{j^*}+K_{j^*}P^*\right). \tag{3.5}$$

Let us now consider several limits of (3.3). The first is when there are no Planck-scale expectation values in the hidden sector. In that case, it is easy to check that eq. (3.3) reduces to the result given in ref. [1, 2],

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left(3T_G-T_R\right)m_{3/2}\,, \tag{3.6}$$

with $b_0 = 3T_G - T_R$.

The second limit is when the Kähler potential is of the "sequestered-sector" form [1],

$$K = -3\log\left[-\frac{1}{3}Q^{+}Q + f(H^{+},H)\right]. \tag{3.7}$$

Here Q and H denote observable and hidden-sector chiral superfields, respectively, and f is a real function with $\langle f\rangle = 1$. This Kähler potential has the property that it generates no tree-level soft scalar masses for the visible-sector fields. It also obeys the relation

$$\frac{1}{d_R}\log\det K|_R{}'' = \frac{1}{3}K \tag{3.8}$$

for vanishing expectation values of the observable fields. (Recall that the derivatives in K'' are taken with respect to visible fields, and the metric is projected on the representation R of dimension d_R of the relevant gauge group.) This implies that the contributions from the Kähler and sigma-model connections cancel, leaving [8]:

$$m_{1/2} = -\frac{g^2}{16\pi^2}(3T_G - T_R)\left(m_{3/2} + \frac{1}{3}K_iF^i\right). \tag{3.9}$$

In the absence of Planck-scale expectation values, this reduces to the result given in refs. [1, 2].

To highlight the importance of the first term in (3.3), it is useful to consider a third example, that of a "no-scale" model, with Kähler potential

$$K = -3\log\left(T + T^{+} - \frac{1}{3}Q^{+}Q - \frac{1}{3}H^{+}H\right). \tag{3.10}$$

This Kähler potential is of the sequestered-sector type (if we treat the modulus T as a hidden-sector field), so the gaugino mass is given by eq. (3.9). If the supersymmetry breaking is dominated by the F component of the modulus T, and the cosmological constant is canceled by adding a constant to the superpotential, it is easy to see that $K_TF^T = -3m_{3/2}$. We see that the anomaly-induced gaugino mass vanishes in this model.

Let us now compare our results with previous work on supergravity anomalies [9, 10]. These papers contain a nonlocal anomaly-induced term that is similar to our expression (2.7), except for the fact that the first term is missing, and the second term is proportional to $(T_G - T_R)\,D^2K$. The expression in these papers has the correct anomalous variation under super-Weyl-Kähler transformations, but it does not transform correctly under super-Weyl or Kähler alone. This can be traced to the missing R^{+} term in their expression for the anomaly. The expressions in these papers completely miss the super-Weyl contribution to the gaugino mass, proportional to $(3T_G - T_R)m_{3/2}$. (This point was also made in [11].) Note that in the "no-scale" model considered above, the R^{+} contribution precisely cancels the terms that involve the Kähler potential.

The nonlocal terms induced by the super-Weyl anomaly are important for a different reason as well. As will be shown in [12], the transformation to the Einstein frame is actually a super-Weyl field redefinition. This redefinition gives rise to an anomalous jacobian in the Einstein-frame lagrangian. The jacobian can be obtained by a shift in (2.7). The jacobian ensures that supersymmetry transformations are not anomalous in the Einstein frame. The issues of frame dependence and the super-Weyl anomaly will be discussed in a separate publication [12].

4. Counterterms from high energies

In this section we will discuss high-energy contributions to the effective lagrangian and their possible effects on anomaly-mediated gaugino masses. These contributions can be generated by heavy modes that are not included in the low-energy supergravity theory.

We first consider contributions that are induced by physics below the Planck scale [8]. Consider, for example, a theory that contains a visible-sector chiral multiplet with a mass of order M, where M is much larger than the scale of the effective field theory. Let us assume that the multiplet gets its mass from symmetry breaking, through a superpotential term of the form

$$\mathcal{L} = \int d^2\Theta\mathcal{E}\,\Phi\, Q^2 + \mathrm{h.c.}\,, \tag{4.1}$$

where Q is in a real representation of the gauge group. In this expression, Φ is a spurion superfield with expectation value $\langle\Phi\rangle = \Phi_0 + F\Theta^2$. Let us include this multiplet in the low-energy theory, and then decouple it by taking $\Phi_0 \to M$. The decoupling is easily done using a field redefinition, $Q \to Q/\sqrt{\Phi/\Phi_0}$, which rotates away the supersymmetry-breaking expectation value, $F\Theta^2$. (This is similar to the D'Hoker-Farhi method for integrating out a heavy top quark [13]. See also [8, 14].) The field redefinition is, of course, anomalous; its jacobian can be computed from the $\log\det K|_R{}''$ part of (2.7):

$$\Delta\mathcal{L} = \frac{g^2}{32\pi^2}\,T_R \int d^2\Theta\, 2\mathcal{E}\log(\Phi/\Phi_0)\, W^\alpha W_\alpha + \mathrm{h.c.} \tag{4.2}$$

As $M \to \infty$, the jacobian induces an additional contribution to the gaugino mass,

$$\Delta m_{1/2} = \frac{g^2}{16\pi^2}\,T_R\,\frac{F}{M}\,. \tag{4.3}$$

This example shows that anomaly-induced gaugino masses can depend on physics beyond the weak scale. In fact, they can also depend on physics beyond the Planck scale. To see this, consider the case of superstring theory, which reduces to supergravity for energies below the Planck scale. If we restrict ourselves to the low-energy supergravity theory, there is no reason for the super-Weyl-Kähler transformations to be anomaly-free. (The low-energy supergravity theory is well defined whether or not the transformations are anomalous.) In string theory, however, it is sometimes necessary to cancel the anomalies.

The low-energy supergravity that arises from superstring compactification typically depends on the size and shape of the extra compact space-time dimensions. In supergravity theory, this information is encoded in the moduli, which are most easily described in terms of sigma models. These sigma models can have various nonlinear symmetries, such as target space duality, which are reflections of exact quantum symmetries of the underlying string theory. These nonlinear symmetries can act on the supergravity theory as super-Weyl-Kähler transformations, in which case the relevant super-Weyl-Kähler anomalies must be canceled.

As a simple example, consider the $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ supersymmetric sigma model, which describes the moduli dynamics in certain toroidal superstring compactifications (see e.g. [15]). The Kähler potential for the (complex) modulus T is given by

$$K = -\log\left(T + T^{+}\right), \tag{4.4}$$

in the usual units with $M_P = 1$. (We neglect matter-field contributions to K.) Target-space duality acts on T through an $\mathrm{SL}(2,\mathbb{R})/\mathbb{Z}_2$ transformation. Under $T \to 1/T$, for example, the Kähler potential changes as follows,

$$K(1/T, 1/T^{+}) = K(T, T^{+}) + \log T + \log T^{+}\,. \tag{4.5}$$

This induces a super-Weyl-Kähler transformation in the low-energy supergravity theory. Now, if duality is an exact quantum symmetry, the corresponding super-Weyl-Kähler anomaly must be canceled by other terms in the low-energy effective action. These terms are local because they arise from integrating out high-energy modes; their variation under the super-Weyl-Kähler transformation precisely cancels the anomaly.

The question for us here is whether the anomaly-canceling counterterms contribute to the gaugino masses. In general, they do, by an amount that depends on unknown high energy physics. Consider, for example, the following term:[4]

$$\Delta\mathcal{L} = \frac{g^2}{16\pi^2}\,(T_G - T_R) \int d^2\Theta 2\mathcal{E} \log\eta(iT)\, W^\alpha W_\alpha + \text{h.c.}\,, \tag{4.6}$$

where $\eta(iT)$ is the Dedekind eta function. This term shifts under $T \to 1/T$, and cancels the super-Weyl-Kähler anomaly arising from (4.5).[5] If T has a Planck-scale

[4] The $\log\eta(iT)$ can be viewed as a contribution to a gauge kinetic function $f(T)$.

[5] The coefficient in $\Delta\mathcal{L}$ can be different when visible-sector matter fields are included.

expectation value, $\langle T \rangle = T_0 + F\Theta^2$, eq. (4.6) gives the following contribution to the gaugino mass

$$\Delta m_{1/2} = -\frac{g^2}{96\pi}\,(T_G - T_R)\,E_2(iT_0)\,F\,, \tag{4.7}$$

where $E_2(iT)$ is the second Eisenstein series.

Other anomaly-canceling terms are also possible. A second example is provided by the Green-Schwarz term from [9],

$$\Delta\mathcal{L} = \frac{g^2}{32\pi^2}\,(T_G - T_R)\int d^2\Theta\, 2\mathcal{E}\,\left(\bar{\mathcal{D}}^2 - 8R\right)K(T^+,T)\,(\Omega - L) + \text{h.c.} \tag{4.8}$$

Here K is the Kähler potential, Ω is the Chern-Simons superfield, and L is a linear multiplet that is required for gauge invariance (for details we refer the reader to [9]). This term shifts the gaugino mass by

$$\Delta m_{1/2} = \frac{g^2}{16\pi^2}\,(T_G - T_R)\,K_T F\,, \tag{4.9}$$

which is clearly not the same as (4.7).

These examples show that the gaugino masses predicted by anomaly meditation are affected by unknown high-energy physics. On the one hand, this makes it difficult to experimentally test the ideas behind anomaly mediation. On the other, the dependence on string-scale physics opens a window to physics at the highest energies. It is an interesting and important problem to study the anomaly-canceling terms in various string vacua.

A final example that illustrates the dependence on high energy physics is the ambiguous separation of the "Kähler potential" and the "superpotential" in the bare lagrangian. At the tree level, the following two superspace lagrangians give rise to the same component lagrangians:

$$\mathcal{L}_0 = \int d^2\Theta\, 2\mathcal{E}\left[\frac{3}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\exp(-K/3) + P\right] + \cdots\,, \tag{4.10}$$

$$\mathcal{L}_0' = \int d^2\Theta\, 2\mathcal{E}\left[\frac{3}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\exp[-(K + \log P + \log P^+)/3] + 1\right] + \cdots\,. \tag{4.11}$$

In these expressions, we have suppressed the gauge-field-dependent terms. At tree-level, eqs. (4.10) and (4.11) are related by a super-Weyl transformation. Beyond tree-level, the equivalence does not persist because of the super-Weyl-Kähler anomaly. The one-loop effective actions derived from these lagrangians are related as follows

$$\mathcal{L}' = \mathcal{L} - \left[\frac{g^2}{32\pi^2}\,(T_G - T_R)\int d^2\Theta\, 2\mathcal{E}\log P\, W^\alpha W_\alpha + \text{h.c.}\right]. \tag{4.12}$$

This relation is easily derived by writing $R' = R - \bar{\mathcal{D}}^2 \log P^+/24$ in (2.7) and then redefining the Kähler potential, $K' = K + \log P + \log P^+$.

Our formula, given in eq. (3.3), was derived using the lagrangian (4.10). If we had used the lagrangian (4.11), we would have found the gaugino mass shifted by[6]

$$\Delta m_{1/2} = -\frac{g^2}{16\pi^2}\,(T_G - T_R)\,\frac{P_i F^i}{P}\,. \tag{4.13}$$

The lagrangian $\mathcal{L}'$ can be made quantum mechanically equivalent to $\mathcal{L}$ by adding an anomaly-canceling counterterm to the bare lagrangian $\mathcal{L}'_0$. Such a counterterm is not necessary for the consistency of the low energy theory, nor is the choice of counterterm unique. Nevertheless, if we add the following counterterm to $\mathcal{L}'_0$,

$$\Delta\mathcal{L}' = \frac{g^2}{32\pi^2}\,(T_G - T_R)\int d^2\Theta\, 2\mathcal{E}\log P\, W^\alpha W_\alpha + \text{h.c.}\,, \tag{4.14}$$

then $\mathcal{L}' + \Delta\mathcal{L}'$ is equivalent to $\mathcal{L}$, and the gaugino mass reduces to (3.3).

5. Conclusions

In this paper we have seen that one-loop anomaly-induced gaugino masses have a natural interpretation in terms of the supersymmetrized Weyl, Kähler and sigma-model anomalies of matter-coupled supergravity. Following Cardoso and Ovrut, we presented the superspace expression for the nonlocal 1PI terms induced by these anomalies. We then found an expression for the visible-sector gaugino mass in terms of the gravitino mass and the hidden-sector auxiliary fields F^i. Our expression holds for arbitrary expectation values. It reduces to previous results when the hidden-sector expectation values are much smaller than $M_{\rm P}$, and in the "sequestered-sector" scenario of Randall and Sundrum [1].

In the last section of the paper, we pointed out that the phenomenology of the anomaly-induced gaugino masses is complicated by possible anomaly-canceling terms in the effective lagrangian. These terms can arise from massive string modes, or from heavy supermultiplets whose masses stem from symmetry breaking. The string-induced terms are present if some exact symmetry of string theory acts by a Kähler transformation on the low-energy supergravity theory. Their form depends on the details of string theory at the Planck scale.

We also pointed out that the two forms of the supergravity action — one where the superpotential P is present, and the other where the superpotential is absorbed in the Kähler potential K — are, in fact, quantum-mechanically inequivalent. In this paper we took as fundamental the action with the explicit superpotential. This

[6] In the sequestered-sector scenario [1], this ambiguity does not arise. In this scenario, there is no direct coupling between hidden- and observable-sector fields in the bare lagrangian. Therefore, one should start with the lagrangian given in eq. (4.10), and not eq. (4.11), since the "Kähler potential" for $\mathcal{L}'_0$ does not satisfy the condition (3.7). For more on anomaly mediation in the five-dimensional supergravity framework, see [16].

action has a geometrical interpretation, in the sense that the superpotential is a section of a holomorphic line bundle over the Kähler manifold [17]. More formal aspects of supergravity anomalies will be investigated in ref. [12].

Finally, let us close by connecting our derivation of the anomaly-induced mass to the discussion presented in the introduction. There, we stated that the gaugino mass follows from the logarithmically-divergent term in the 1PI effective action,

$$\frac{1}{4}\int d^2\theta\left(1-\frac{g^2 b_0}{16\pi^2}\log\frac{\Lambda^2}{\Box}\right)W^\alpha W_\alpha+\text{h.c.} \tag{5.1}$$

where $b_0 = 3T_G - T_R$. This expression assumes that all hidden-sector expectation values are much smaller than the Planck scale, and that spacetime is flat and Minkowski. If we interpret the cutoff as a supersymmetry-breaking spurion, $\Lambda \to \Lambda(1+\theta^2 m_{3/2})$, eq. (5.1) gives rise to the nonvanishing gaugino mass (1.3).

Suppose now that we choose another cutoff, perhaps one that preserves supersymmetry. How then can we find the anomaly-induced gaugino mass? The answer is to write (5.1) in a weak supergravity background. In particular, we take the background to be superconformally flat, with $E_m{}^a = \eta_m{}^a\, e^{\Sigma+\Sigma^+}$, where Σ is a θ-dependent but x-independent conformal factor. We replace the flat-space laplacian by $\Box\, e^{-4\Sigma}$, which is the relevant part of the covariant, chiral, curved-space laplacian in the superconformal background [18]. This gives

$$\begin{aligned}\frac{1}{4}\int d^2\theta&\left(1-\frac{g^2 b_0}{16\pi^2}\left[\log\left(\frac{\Lambda^2}{\Box}e^{4\Sigma}\right)\right]\right)W^\alpha W_\alpha+\text{h.c.} = \\ &=\frac{1}{4}\int d^2\theta\left(1-\frac{g^2 b_0}{16\pi^2}\left[\log\left(\frac{\Lambda^2}{\Box}\right)+4\Sigma\right]\right)W^\alpha W_\alpha+\text{h.c.}\end{aligned} \tag{5.2}$$

In the superconformal background, $R = -\bar{D}^2\Sigma^+/4 + \cdots$, so this becomes

$$\frac{1}{4}\int d^2\theta\left(1-\frac{g^2 b_0}{16\pi^2}\left[\log\left(\frac{\Lambda^2}{\Box}\right)-\frac{\bar{D}^2}{\Box}R^+\right]\right)W^\alpha W_\alpha+\text{h.c.} \tag{5.3}$$

plus higher-order terms. The term proportional to R^+ is nothing but the Weyl anomaly term from (2.7).

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